Trigonométrie

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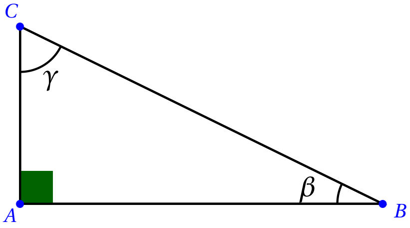
Cosinus, sinus, tangente d'un angle, définition

Dans un triangle ABC rectangle en A, on définit le cosinus (cos), le sinus (sin) et la tangente (tan) de l'angle $\beta = \widehat{ABC}$ ou $\gamma = \widehat{BCA}$:



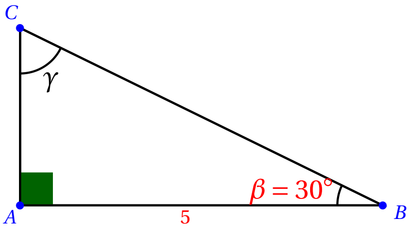
$$\cos(\beta) = \frac{AB}{BC} \qquad \cos(\gamma) = \frac{AC}{BC}$$

$$\sin(\beta) = \frac{AC}{BC} \qquad \sin(\gamma) = \frac{AB}{BC}$$

$$\tan(\beta) = \frac{AC}{AB} \qquad \tan(\gamma) = \frac{AB}{AC}$$

Cosinus, sinus, tangente d'un angle, exemples

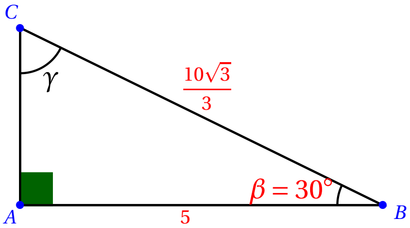
Dans un triangle ABC rectangle en A, on donne $AB = 5$ et $\beta = 30°$



$$\cos(\beta) = \frac{AB}{BC} \Longleftrightarrow BC = \frac{AB}{\cos(\beta)} = \frac{5}{\cos(30)} = \frac{5}{\frac{\sqrt{3}}{2}} = \frac{10}{\sqrt{3}} = \frac{10\sqrt{3}}{3}$$

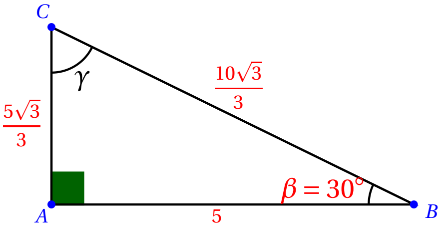
Cosinus, sinus, tangente d'un angle, exemples

Dans un triangle ABC rectangle en A, on donne $AB = 5$ et $\beta = 30°$



$$\sin(\beta) = \frac{AC}{BC} \iff AC = BC \times \sin(\beta) = \frac{10\sqrt{3}}{3} \times \sin 30 = \frac{10\sqrt{3}}{3} \times \frac{1}{2} = \frac{5\sqrt{3}}{3}$$

Dans un triangle ABC rectangle en A, on donne $AB = 5$ et $\beta = 30°$

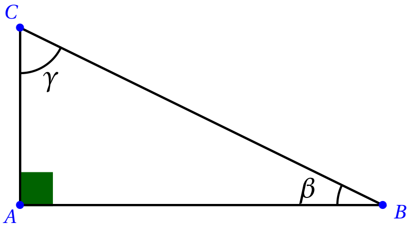


$$\tan(\gamma) = \frac{AB}{AC} \iff \tan(\gamma) = \frac{5}{\frac{5\sqrt{3}}{3}} = 5 \times \frac{3}{5\sqrt{3}} = \frac{3}{\sqrt{3}} = \sqrt{3}$$

$$\gamma = \tan^{-1}(\sqrt{3}) = 60$$

Cosinus, sinus, tangente d'un angle, propriétés

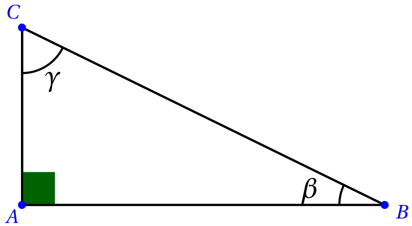
Dans un triangle ABC rectangle en A, on définit le cosinus (cos), le sinus (sin) et la tangente (tan) de l'angle $\beta = \widehat{ABC}$ ou $\gamma = \widehat{BCA}$:



$$\cos^2(\beta)+\sin^2(\beta) = \left(\frac{AB}{BC}\right)^2 + \left(\frac{AC}{BC}\right)^2 = \frac{AB^2}{BC^2} + \frac{AC^2}{BC^2} = \frac{AB^2 + AC^2}{BC^2} = \frac{BC^2}{BC^2} = 1$$

Cosinus, sinus, tangente d'un angle, propriétés

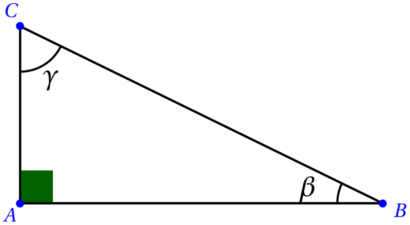
Dans un triangle ABC rectangle en A, on définit le cosinus (cos), le sinus (sin) et la tangente (tan) de l'angle $\beta = \widehat{ABC}$ ou $\gamma = \widehat{BCA}$:



$$\tan(\beta) = \frac{AC}{AB} = \frac{\frac{AC}{BC}}{\frac{AB}{BC}} = \frac{\sin(\beta)}{\cos(\beta)}$$

Cosinus, sinus, tangente d'un angle, propriétés

Dans un triangle ABC rectangle en A, on définit le cosinus (cos), le sinus (sin) et la tangente (tan) de l'angle $\beta = \widehat{ABC}$ ou $\gamma = \widehat{BCA}$:

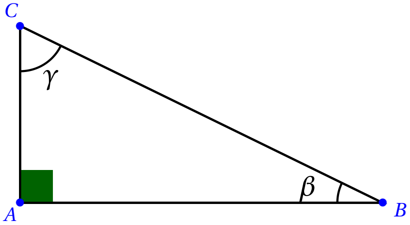


$$\cos^2(\beta) + \sin^2(\beta) = 1 \text{ et } \tan(\beta) = \frac{\sin(\beta)}{\cos(\beta)}$$

$$1+\tan^2(\beta) = 1+\left(\frac{\sin(\beta)}{\cos(\beta)}\right)^2 = 1+\frac{\sin^2(\beta)}{\cos^2(\beta)} = \frac{cos^2(\beta)+\sin^2(\beta)}{\cos^2(\beta)} = \frac{1}{\cos^2(\beta)}$$

Cosinus, sinus, tangente d'un angle, propriétés

Dans un triangle ABC rectangle en A, on définit le cosinus (cos), le sinus (sin) et la tangente (tan) de l'angle $\beta = \widehat{ABC}$ ou $\gamma = \widehat{BCA}$:



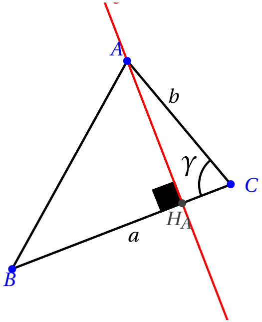
$$\cos^2(\beta) + \sin^2(\beta) = 1$$

$$\tan(\beta) = \frac{\sin(\beta)}{\cos(\beta)}$$

$$1 + \tan^2(\beta) = \frac{1}{\cos^2(\beta)}$$

Trigonométrie et aire d'un triangle

Soit ABC un triangle tel que $a = BC$, $b = AC$ et $c = AB$, $\gamma = \widehat{ACB}$.
Soit la hauteur issue de A et H_A son pied.



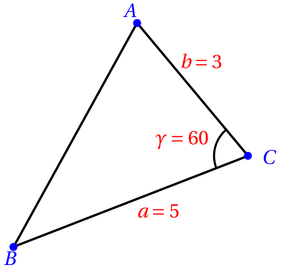
L'aire $\mathscr{A}$ du triangle ABC en fonction de a et AH_A est
$\mathscr{A} = \dfrac{1}{2} \times a \times AH_A$.

$\sin(\gamma) = \dfrac{AH_A}{AC} \Longleftrightarrow AH_A = \sin(\gamma) \times AC = b\sin(\gamma)$

Ainsi $\mathscr{A} = \dfrac{1}{2} ab\sin(\gamma)$.

Trigonométrie et aire d'un triangle exemple

Soit ABC un triangle tel que $a = BC$=5, $b = AC = 3$ et $c = AB$, $\gamma = \widehat{ACB} = 60°$.



L'aire $\mathscr{A}$ du triangle ABC est

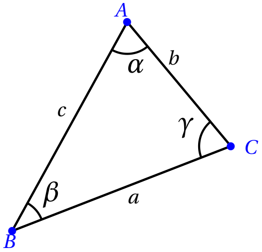
$$\mathscr{A} = \frac{1}{2}ab\sin(\gamma) = \frac{1}{2} \times 5 \times 3 \times \sin(60) = \frac{15}{2} \times \frac{\sqrt{3}}{2} = \frac{15\sqrt{3}}{4}$$

Trigonométrie et aire d'un triangle, propriété

Soit ABC un triangle tel que $a = BC$, $b = AC$ et $c = AB$, $\gamma = \widehat{ACB}$.

On note $\mathscr{A}$ l'aire du triangle ABC, on a $\mathscr{A} = \dfrac{1}{2}ab\sin(\gamma)$.

$$\frac{abc}{2\mathscr{A}} = \frac{abc}{2 \times \frac{1}{2}ab\sin(\gamma)} = \frac{c}{\sin(\gamma)}$$



$$\frac{abc}{2\mathscr{A}} = \frac{a}{\sin(\alpha)} = \frac{b}{\sin(\beta)} = \frac{c}{\sin(\gamma)}$$

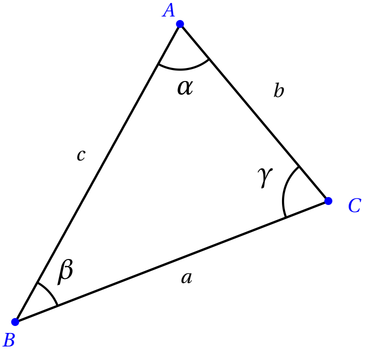
Relations métriques : Al-Kashi

ABC est un triangle,
$\alpha = \widehat{BAC}$, $\beta = \widehat{ABC}$ et $\gamma = \widehat{ACB}$,
$a = BC$, $b = AC$ et $c = AB$.

$$a^2 = b^2 + c^2 - 2bc\cos(\alpha)$$

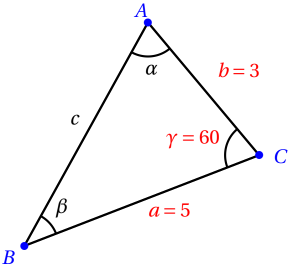
$$b^2 = a^2 + c^2 - 2ac\cos(\beta)$$

$$c^2 = b^2 + a^2 - 2ba\cos(\gamma)$$



Relations métriques : Al-Kashi, exemple

ABC est un triangle,
$\gamma = \widehat{ACB} = 60$,
$a = BC = 5$, $b = AC = 3$.



$c^2 = b^2 + a^2 - 2ba\cos(\gamma)$
$c^2 = 3^2 + 5^2 - 2 \times 3 \times 5 \times \cos(60) = 34 - 2 \times 15 \times \dfrac{1}{2} = 19.$
$c = \sqrt{19}.$

Relations métriques : Al-Kashi, exemple

ABC est un triangle,
$\gamma = \widehat{ACB} = 60$,
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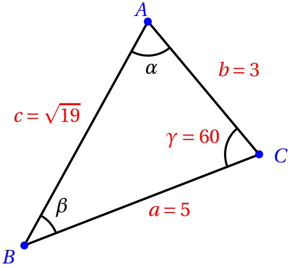


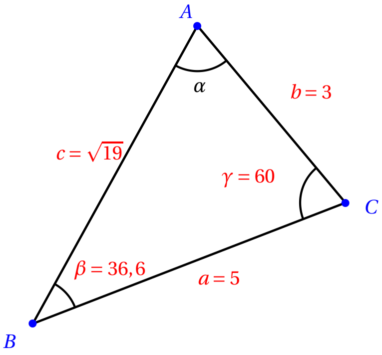
$$b^2 = a^2 + c^2 - 2ac\cos(\beta)$$

$$\cos(\beta) = \frac{a^2+c^2-b^2}{2ac} = \frac{5^2+\sqrt{19}^2-3^2}{2\times5\times\sqrt{19}} = \frac{7}{2\sqrt{19}} = \frac{7\sqrt{19}}{38}.$$

$$\beta = \cos^{-1}\left(\frac{7\sqrt{19}}{38}\right) \simeq 36,6$$

Relations métriques : Al-Kashi, exemple

ABC est un triangle,
$\gamma = \widehat{ACB} = 60$,
$a = BC = 5$, $b = AC = 3$.



$\alpha \simeq 180 - (60 + 36,6) = 83,4$.

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