Produit scalaire
Al Kashi exemples

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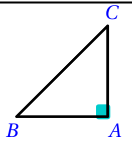
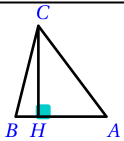
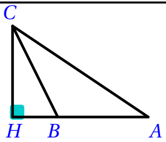
Théorème d'Al Kashi,trigonométrie dans un triangle



$$a^2 = b^2 + c^2 = b^2 + c^2 - 2bc\cos(90)$$
$$= b^2 + c^2 - 2b.c.\cos\left(\widehat{CBA}\right)$$



$$a^2 = BH^2 + HC^2$$
$$= (c - AH)^2 + b^2 - AH^2$$
$$= b^2 + c^2 - 2c.AH$$
$$= b^2 + c^2 - 2c.b.\cos\left(\widehat{CBA}\right)$$



$$a^2 = BH^2 + HC^2$$
$$= (AH - c)^2 + b^2 - AH^2$$
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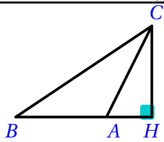
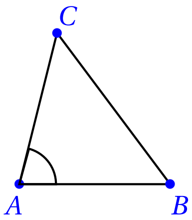
$$= b^2 + c^2 - 2c.b.\cos\left(\widehat{CAB}\right)$$



$$a^2 = BH^2 + HC^2$$
$$= (c + AH)^2 + b^2 - AH^2$$
$$= b^2 + c^2 + 2c.AH$$
$$= b^2 + c^2 + 2c.b.\cos\left(\widehat{HAC}\right)$$
$$= b^2 + c^2 + 2c.b.\cos\left(180 - \widehat{CAB}\right)$$
$$= b^2 + c^2 - 2c.b.\cos\left(\widehat{CAB}\right)$$

Théorème d'Al Kashi, produit scalaire

Soit ABC un triangle quelconque, on note $a = BC$, $b = AC$ et $c = AB$:



$a^2 = \overrightarrow{BC}^2 = \left(\overrightarrow{BA} + \overrightarrow{AC}\right)^2 = \left(\overrightarrow{AC} - \overrightarrow{AB}\right)^2 = \|\overrightarrow{AC}\|^2 - 2.\overrightarrow{AC}.\overrightarrow{AB} + \|\overrightarrow{AB}\|^2 =$
$AC^2 + AB^2 - 2.AC.AB.\cos\left(\widehat{BAC}\right)$.

Ainsi : $a^2 = b^2 + c^2 - 2bc\cos\left(\widehat{BAC}\right)$.

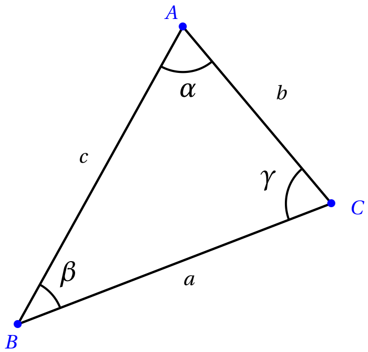
Théorème d'Al Kashi

ABC est un triangle,
$\alpha = \widehat{BAC}$, $\beta = \widehat{ABC}$ et $\gamma = \widehat{ACB}$,
$a = BC$, $b = AC$ et $c = AB$.

$$a^2 = b^2 + c^2 - 2bc\cos(\alpha)$$

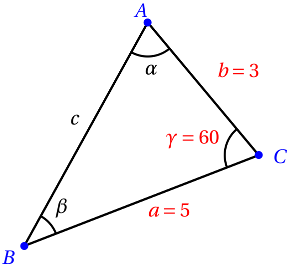
$$b^2 = a^2 + c^2 - 2ac\cos(\beta)$$

$$c^2 = b^2 + a^2 - 2ba\cos(\gamma)$$



Relations métriques : Al-Kashi, exemple

ABC est un triangle,
$\gamma = \widehat{ACB} = 60$,
$a = BC = 5$, $b = AC = 3$.



$c^2 = b^2 + a^2 - 2ba\cos(\gamma)$

$c^2 = 3^2 + 5^2 - 2 \times 3 \times 5 \times \cos(60) = 34 - 2 \times 15 \times \dfrac{1}{2} = 19.$

$c = \sqrt{19}.$

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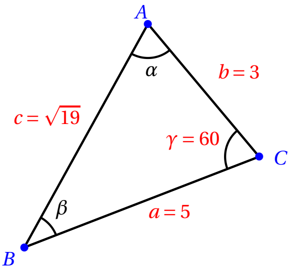

$$b^2 = a^2 + c^2 - 2ac\cos(\beta)$$

$$\cos(\beta) = \frac{a^2 + c^2 - b^2}{2ac} = \frac{5^2 + \sqrt{19}^2 - 3^2}{2 \times 5 \times \sqrt{19}} = \frac{7}{2\sqrt{19}} = \frac{7\sqrt{19}}{38}.$$

$$\beta = \arccos\left(\frac{7\sqrt{19}}{38}\right) \simeq 36,6$$

Relations métriques : Al-Kashi, exemple

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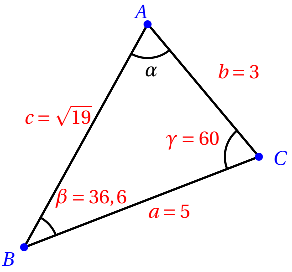

$\alpha \simeq 180 - (60 + 36,6) = 83,4$.
ou
$a^2 = b^2 + c^2 - 2bc\cos(\alpha)$
$\cos(\alpha) = \dfrac{b^2 + c^2 - a^2}{2bc} = \dfrac{3^2 + \sqrt{19}^2 - 5^2}{2 \times 3 \times \sqrt{19}} = \dfrac{1}{2\sqrt{19}} = \dfrac{\sqrt{19}}{38}$.
$\beta = \arccos\left(\dfrac{\sqrt{19}}{38}\right) \simeq 83,4$

FIN